# Lifetime in Storage Rings

Fernando Sannibale

# Introduction

• Particles are lost in accelerators because of the accelerator finite aperture.

• Many processes can excite particles on orbits larger than the nominal. If the displacement in the new orbit is larger than the aperture, the particle is obviously lost.

• The limiting aperture in accelerators can be *physical* or *dynamic*. The vacuum chamber defines the physical aperture, while transverse and momentum acceptances of the accelerator define the dynamical one.

• Processes important for the lifetime include: *elastic and inelastic residual gas scattering*, *scattering with the other particles in the beam*, *quantum lifetime for electrons and positrons*, *tune resonances*, …

• Damping plays a major role in the electron/positron case. For protons and heavy ions, lifetime is usually much longer but any perturbation will progressively build-up and generate losses.

• For most applications beam needs to be stored for as long as possible, so it is very important to contain the above effects within acceptable values.

• Such a requirement has important consequences on the design constraints. For example, limiting the effects of the residual gas scattering pushes towards ultra high vacuum technologies.

# The Concept of Lifetime

- **In a loss process, the number of particles lost at the time $t$ is proportional to the number of particles present in the beam at the time $t$:**

$$dN = -\alpha\, N(t)\, dt \qquad with \quad \alpha \equiv constant$$

**By defining the lifetime $\tau$ as:** $\boxed{\tau = \dfrac{1}{\alpha}}$ $\Rightarrow$ $\boxed{N = N_0 e^{-t/\tau}}$

- **From the last equation, one can see that the lifetime is defined as the time required for the beam to reduce its number of particles to $1/e$ of the initial value.**

- **Lifetime due to the individual effects (gas, Touschek, …) can be similarly defined. The total lifetime will be then obtained by summing the individual contributions:**

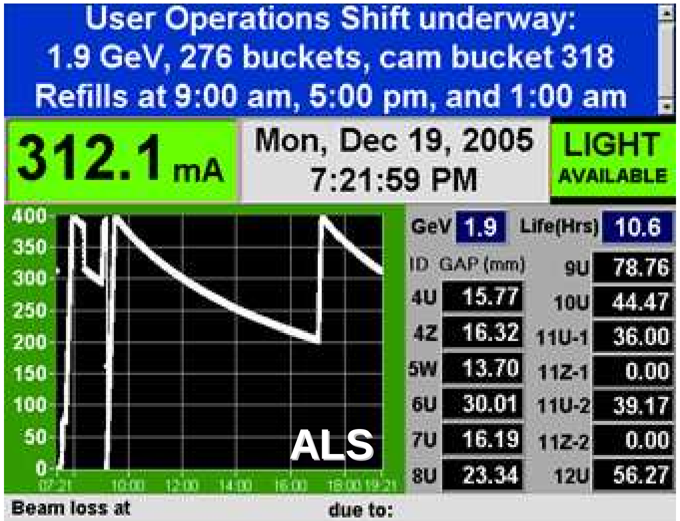
$$\frac{1}{\tau} = \frac{1}{\tau_1} + \frac{1}{\tau_2} + \frac{1}{\tau_3} + \ldots.$$

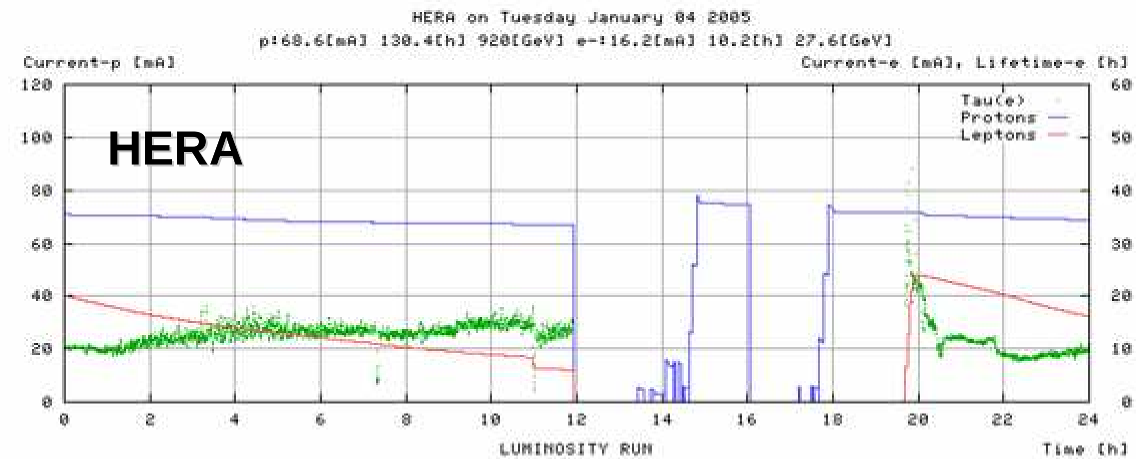
- **With this definition, the problem of calculating the lifetime is reduced to the evaluation of the single lifetime components.**

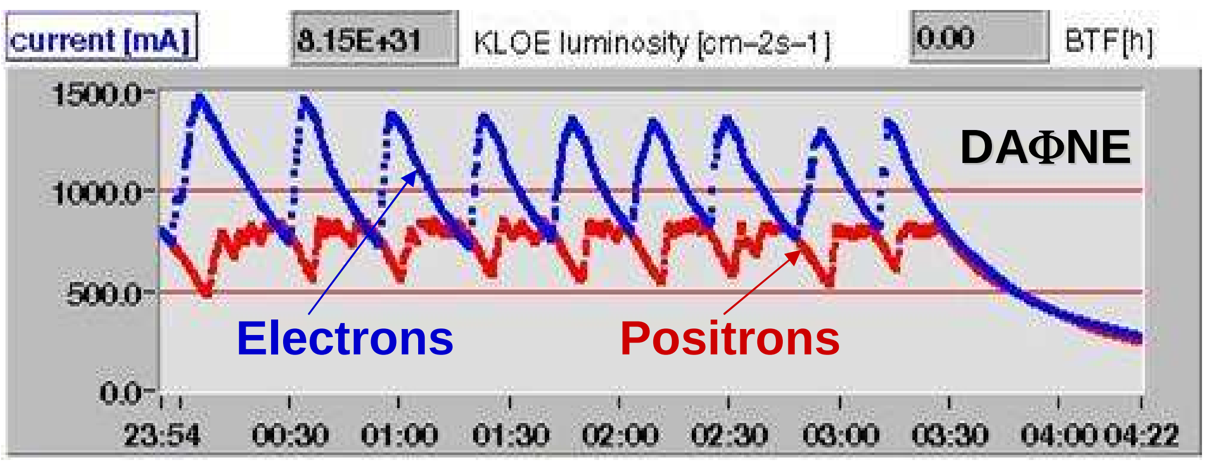
# Is the Constant Lifetime Model Accurate?

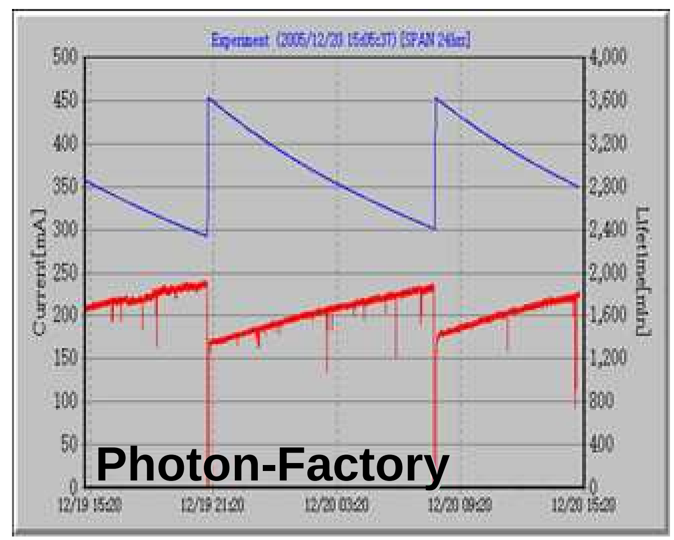
- **The previous model, where the lifetime was assumed constant, is often too simple for describing the case of real accelerators.**
- **In fact, in most of the electron storage rings the lifetime actually depends on current.**
- **In fact, the *Touschek effect* (discussed later), whose contribution dominates the losses in many of the present electron accelerators, depends on current. When the stored current decreases with time, the losses due to Touschek decrease as well and the lifetime increases.**
- **Additionally, synchrotron radiation by hitting the vacuum chamber transfers the energy required to the molecules trapped in the vacuum chamber wall to be released (*gas desorption*).**
- **Because of this, for higher stored currents, the synchrotron radiation intensity increases generating more desorption and increasing the pressure in the vacuum chamber (*dynamic pressure*). This will increase the scattering of the beam with the residual gas, with a consequent reduction of the beam lifetime.**
- **Anyway, for reasonably small variations of the current, the constant lifetime assumption is locally valid and it is widely used.**

# Example of Lifetime in Real Accelerators



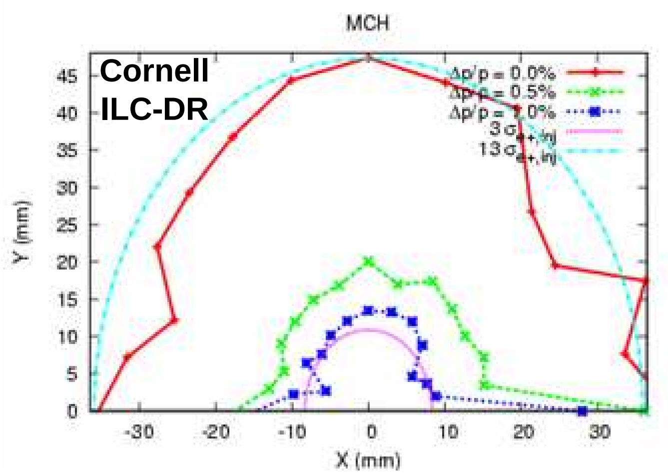
# Dynamic Aperture and Momentum Acceptance

- **Quite often in existing storage rings, the aperture is not limited by the vacuum chamber size.**

- **In fact, nonlinearities in the fields of the magnets create resonance “islands” in the phase space that can capture particles with large amplitude orbits and bring them in collision with the vacuum chamber.**

- **This effect creates a “virtual” aperture for the machine which is usually referred as the dynamic aperture**

- **Due to their strong nonlinear nature, dynamic apertures can be calculated only numerically.**




- **In the longitudinal plane, the *momentum acceptance* is limited by the size of the RF bucket or by the dynamic aperture for the off-momentum particles. In fact, off-energy particles in dispersive regions can hit the dynamic aperture of the ring even if their momentum difference is still within the limits of the RF acceptance.**

# Cross Section of a Scattering Event

- **In scattering processes it is useful to define as *cross section $\sigma$ the event rate per unit incident flux and per target particle*.**

- **Let us consider two groups of particles. Particles in the same group have same momentum and are distributed in uniform spatial distributions.**

- **For an interaction with cross section $\sigma$, the number of events per second (event rate), in the rest frame of the particle group 2 for example, is given by:**

$$\dot{N} = \frac{dN}{dt} = \phi_1 N_2 \sigma = (n_1 v_1)(n_2 V_{IR})\sigma = n_1 n_2 v_1 V_{IR} \sigma$$

**where $n_1$ and $n_2$ are the densities of the two groups of particles $v_1$ is the velocity of the particle group 1 and $V_{IR}$ is the volume of the region where the two particles interact.**

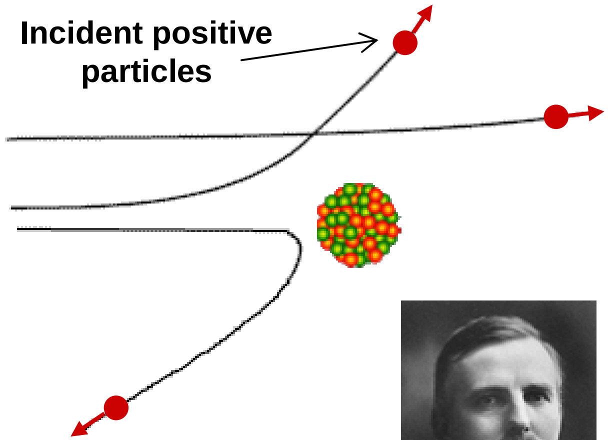
- **The equation above applies for uniform densities $n_1$, $n_2$ and constant $\sigma$. For the more general case where these quantities depend on position, the above expression must be replaced with:**

$$\dot{N} = v_1 \int_{V_{IR}} \sigma \, n_1 \, n_2 \, dx \, dy \, dz$$

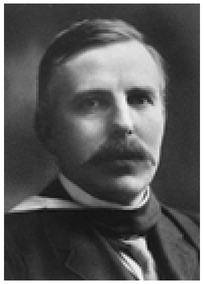
# Gas Lifetime: Elastic Scattering

• When a charged particle passes close to a residual gas molecule, it is deflected by the electric field of the molecule nucleus.

• This phenomenon is a particular case of *Coulomb scattering* and it is usually referred as Rutherford scattering, after the name of the English scientist that first discovered it in 1911.




• Rutherford experiments were quite important because proved that the atom mass is not uniformly distributed (Thomson model) but instead concentrated in a very small positively charged part of the atom, the nucleus.



• The equation of motion for the problem can be solved showing that the trajectories of the scattered particles are hyperbolae (Kepler problem).

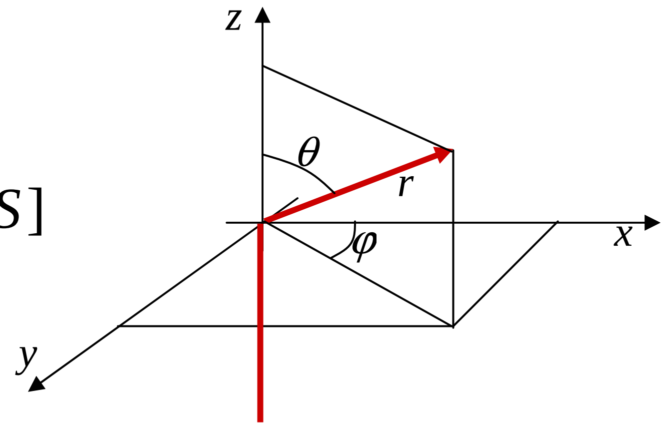
• In the process, the incident particle does not loose energy, so this kind of scattering is referred as *elastic*.

• In a storage ring, when a beam particle scatters with a residual gas molecule it undergoes to betatron oscillations. If the oscillation amplitude is larger than the ring acceptance the particle is lost.

# Rutherford Scattering Cross Section

- **Rutherford calculated the differential cross section for the elastic scattering of a charged particle with a nucleus:**

$$\frac{d\sigma_R}{d\Omega}=\frac{1}{(4\pi\varepsilon_0)^2}\left(\frac{Z_{Inc}Ze^2}{2\beta c\, p}\right)^2\frac{1}{\sin^4(\theta/2)}\quad [MKS]$$




**where $Z_{Inc}\, e$ is the charge of the incident particle, $Ze$ is the charge of the nucleus, $\beta c$ and $p$ are the velocity and the momentum of the incident particles and $\theta$ is the scattering angle. $\Omega$ is the solid angle.**

- **In deriving the previous equation, screening effects of the atom electrons and direct inelastic scattering with the atoms electrons were neglected because small. Nucleus recoil has been neglected as well.**

- **For small angles, the screening from the molecule electrons must be taken into account and for large scattering the nucleus finite size must be considered.**

# Gas Lifetime: Elastic Scattering

- In the case of a beam of $N$ particles scattering on a residual gas molecule in a storage ring, the accelerator aperture will limit the scattered angle to some value $\theta_{MAX}$. For scattered angles larger than $\theta_{MAX}$ the particle will be lost.
- By using the definition of cross section, the rate of losses is given by:

$$\left.\frac{dN}{dt}\right|_{Gas} = -\phi_{beam\ particles} N_{molecules} \sigma_R$$

- If $n$ is the gas molecule density, $A_T$ the beam transverse size, $L$ the ring length, $T$ the revolution period and $\beta c$ the beam velocity, then:

$$\phi_{beam\ particles} = \frac{N}{A_T T} = \frac{N}{A_T}\frac{\beta c}{L} \qquad N_{molecules} = n A_T L \qquad \sigma_R = \int_{Lost} \frac{d\sigma_R}{d\Omega} d\Omega = \int_0^{2\pi} d\varphi \int_{\theta_{MAX}}^{\pi} \frac{d\sigma_R}{d\Omega} \sin\theta \, d\theta$$

$$\left.\frac{dN}{dt}\right|_{Gas} = -\frac{\pi n N \beta c}{2(4\pi\varepsilon_0)^2}\left(\frac{Z_{Inc} Z e^2}{\beta c p}\right)^2 \int_{\theta_{MAX}}^{\pi} \frac{\sin\theta \, d\theta}{\sin^4 \theta/2}$$

$$\boxed{\left.\frac{dN}{dt}\right|_{Gas} = -\frac{\pi n N \beta c}{(4\pi\varepsilon_0)^2}\left(\frac{Z_{Inc} Z e^2}{\beta c p}\right)^2 \frac{1}{\tan^2(\theta_{MAX}/2)}}$$

*Loss rate for gas elastic scattering [MKS]*

# Gas Lifetime: Elastic Scattering

- **The number of molecules for cubic centimeter, for a gas at 0 ºC and at 760 Torr, is given by the *Loschmidt's constant* $n_0$ = 2.68675 x $10^{25}$ $m^{-3}$**

- **If we assume that our gas is composed by *M*-atomic molecules and that its pressure is *P*, then the density of the gas is:**

$$n = M\, n_0 \frac{P_{[Torr]}}{760}$$

- **For a ring with acceptance $\varepsilon_A$ and for small $\theta$, the maximum scattering angle at the scattering point is:**

$$\theta_{MAX}(s) = \sqrt{\frac{\varepsilon_A}{\beta_T(s)}}$$

- **For an <u>estimate</u>, we can replace $\beta_T$ with its average value along the ring:**

$$\langle \theta_{MAX} \rangle = \sqrt{\frac{\varepsilon_A}{\langle \beta_T \rangle}}$$

- **Which used with the previous results gives:**

$$\left.\frac{dN}{dt}\right|_{Gas} \cong -\frac{\pi M n_0 \beta c\, N}{(4\pi\varepsilon_0)^2} \frac{P_{[Torr]}}{760} \left(\frac{Z_{Inc} Z e^2}{\beta c\, p}\right)^2 \frac{4\langle \beta_T \rangle}{\varepsilon_A}$$

**That integrated:** $N = N_0 \exp(-t/\tau_{Gas})$

**with:**

$$\boxed{\tau_{Gas} \cong \frac{760}{P_{[Torr]}} \frac{4\pi\varepsilon_0^2}{\beta c\, M\, n_0} \left(\frac{\beta c\, p}{Z_{Inc} Z e^2}\right)^2 \frac{\varepsilon_A}{\langle \beta_T \rangle} \quad [MKS]}$$

# Gas Lifetime: Inelastic Scattering

• **In the inelastic scattering, the incident particles lose energy during the process.**

• **We can distinguish two main phenomena:**

• ***Gas bremsstrahlung*: the incident particle is deflected by the molecule nucleus and because of the transverse acceleration, radiates a photon. This effect is important for relativistic particles**

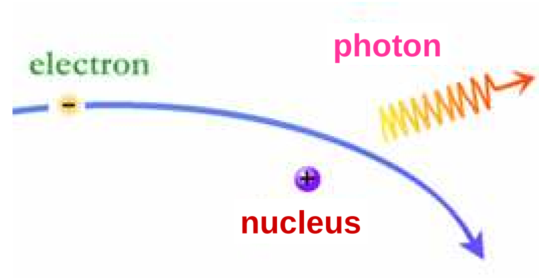
• ***Atom excitation*: the interaction brings the atom to ionization or into an excited state. The effect is important for non relativistic particles**

• **For both the processes, if the amount of lost energy is beyond the momentum acceptance of the ring the particle is lost.**

• **The lifetime contribution due to inelastic scattering is calculated following the same steps used for the elastic case, replacing the cross section for the elastic scattering with the sum of the two cross-section terms for the inelastic case.**

# Gas Bremsstrahlung

- **Bremsstrahlung in accelerators is important for relativistic electrons and positrons.**




  - **The differential cross-section was first calculated by Bethe and Heitler:**

$$\frac{d\sigma_B}{d\kappa}=\alpha\frac{Z^2r_0^2}{\kappa}\left\{\left[\frac{4}{3}\left(1-\frac{\kappa}{E_0}\right)+\left(\frac{\kappa}{E_0}\right)^2\right]\left[\varphi_1(\kappa_1)-\frac{4}{3}\ln Z\right]+\frac{2}{3}\left(1-\frac{\kappa}{E_0}\right)\left[\varphi_2(\kappa_1)-\varphi_1(\kappa_1)\right]\right\}$$

**Where:** $r_0=2.818\times10^{-15}\,m\equiv classical\ electron\ radius$

$$\alpha=\frac{e^2}{4\pi\varepsilon_0 hc}=\frac{1}{137}\equiv fine\ structure\ constant$$

$$\kappa\equiv photon\ energy\equiv energy\ lost\ by\ the\ particle$$

$$E_0\equiv particle\ energy$$

$$\varphi_1,\varphi_2\equiv screening\ functions$$

**with:** $$\kappa_1=100\frac{\kappa}{Z^{1/3}}\frac{m_0c^2}{E_0(E_0-\kappa)}$$

- **For high relativistic electrons, the screening is maximum and the cross-section becomes (*complete screening case*):**

$$\frac{d\sigma_B}{d\kappa}=\alpha\frac{4Z^2r_0^2}{\kappa}\left\{\left[\frac{4}{3}\left(1-\frac{\kappa}{E_0}\right)+\left(\frac{\kappa}{E_0}\right)^2\right]\left[5.209-\frac{1}{3}\ln Z\right]+\frac{1}{9}\left(1-\frac{\kappa}{E_0}\right)\right\}$$

# Gas Bremsstrahlung

- After some manipulations, a simple expression for the lifetime contribution due to gas bremsstrahlung can be obtained:

$$\frac{1}{\tau_{Brem}} = -\frac{4}{3}\frac{c}{L_R}\ln\left(\frac{\Delta E_A}{E_0}\right)$$

$$\frac{1}{L_R} \cong r_0^2 Z^2 \alpha\, n\left(\frac{2}{9} + 4\ln\frac{183}{Z^{1/3}}\right)$$

- Here, $\Delta E_A$ is the *energy acceptance* of the storage ring. $L_R$ is radiation length of the gas and is defined as the length required to the particle to lose $(1 - e^{-1})$ of its energy when traveling trough the gas.

- In a real accelerator, the residual gas is a combination of different molecular species. Anyway, it turns out that the average $\langle Z^2 \rangle$ over the different species is ~ 50 which is approximately the value for nitrogen.

- This allow to write with good approximation:

$$\tau_{Brem[hours]} \cong -\frac{153.14}{\ln(\Delta E_A / E_0)}\frac{1}{P_{[nTorr]}}$$

# Atomic Excitation

- **The differential cross-section for the atomic excitation contribution is very similar to the one for the bremsstrahlung:**

$$\frac{d\sigma_{Exc.}}{d\kappa}=\alpha\frac{Z\,r_0^2}{\kappa}\left\{\left[\frac{4}{3}\left(1-\frac{\kappa}{E_0}\right)+\left(\frac{\kappa}{E_0}\right)^2\right]\left[\psi_1(\kappa_2)-\frac{8}{3}\ln Z\right]+\frac{2}{3}\left(1-\frac{\kappa}{E_0}\right)\left[\psi_2(\kappa_2)-\psi_1(\kappa_2)\right]\right\}$$

$Z\ \ instead\ of\ \ Z^2$ $\qquad$ $8\ \ instead\ of\ \ 4$

**with different:** $\psi_1,\psi_2\equiv screening\ \ functions$ **and:** $\kappa_2=100\frac{\kappa}{Z^{2/3}}\frac{m_0c^2}{E_0(E_0-\kappa)}$

- **For the electron accelerator case, these differences make the cross-section for atomic excitation much smaller than the bremsstrahlung one.**

- **For extremely relativistic particles, the complete screen case gives:**

$$\frac{d\sigma_{Exc}}{d\kappa}=\alpha\frac{4Zr_0^2}{\kappa}\left\{\left[\frac{4}{3}\left(1-\frac{\kappa}{E_0}\right)+\left(\frac{\kappa}{E_0}\right)^2\right]\left[7.085-\frac{2}{3}\ln Z\right]+\frac{1}{9}\left(1-\frac{\kappa}{E_0}\right)\right\}$$

# Gas Lifetime: Example of Vacuum Requirements

- We already saw that for <u>electrons</u> in the approximation of $\langle Z^2 \rangle \sim 50$, we have for the gas bremsstrahlung lifetime:

$$\tau_{Brem[hours]} \cong -\frac{153.14}{\ln\left(\Delta E_A / E_0\right)} \frac{1}{P_{[nTorr]}}$$

- In the same approximation, the inelastic gas scattering lifetime becomes:

$$\tau_{Gas[hours]} \cong 10.25 \frac{E^2_{0\,[GeV]}}{P_{[nTorr]}} \frac{\varepsilon_{A[\mu m]}}{\left\langle \beta_T \right\rangle_{[m]}}$$

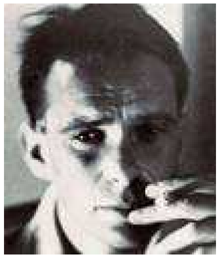
- Evaluating these expressions for the typical electron ring case, one finds that the requirement on vacuum is for <u>dinamic</u> pressures of the order of the nTorr.

# Touschek Effect

• Particles in the bunch are subjected to betatron oscillations. Coulomb scattering between the particles can transfer transverse momentum to the longitudinal plane.

• If this extra momentum brings the two scattered particles beyond the momentum acceptance of the ring, then the particles are lost.

• This process is usually referred as the **Touschek effect** after the Austrian scientist that discovered it.



1921-1978


• The first observation was done in the early 60's in Frascati at ADA, the electron-positron accelerator conceived by Touschek and the first ever built.

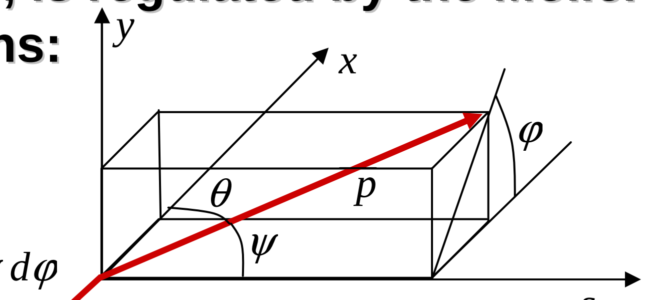

• The Touschek is the dominant effect limiting the lifetime in many of the modern electrons storage rings.

# Touschek Effect

- The effect can be properly investigated in the center of mass system (CMS), where the particles are non-relativistic.
- In this frame, the Coulomb scattering between two particles of the same specie and with equal but opposite momentum ***p***, is regulated by the Möller differential cross-sections:

$$\frac{d\sigma_{Tousc.}}{d\Omega} = \frac{4r_0^2}{\beta^2}\left[\frac{4}{\sin^4\theta} - \frac{3}{\sin^2\theta}\right]$$

$$\beta c \equiv CMS \quad velocity$$




$$d\Omega = \sin\psi \, d\psi \, d\varphi$$

$$\cos\theta = \sin\psi \cos\varphi$$

- In the CMS, the longitudinal component of the momentum due to the scattering is:

$$p_S \equiv p\cos\psi$$

- Which in the laboratory system becomes:

$$\boxed{p'_S = \gamma\left(p_S - \frac{\beta}{c}E\right) \sim \gamma\, p_S = \gamma\, p\cos\psi}$$

where the ~ sign is a good approximation because the particles are non-relativistic in the CMS.

- The last equation shows how the momentum transfer in the laboratory system is <u>amplified by a factor $\gamma$</u>.

# Touschek Effect

- If $\gamma p_s$ is larger than the momentum acceptance $\Delta p_A$, both the scattered particles are lost. And the condition for losing a particle becomes:

$$\left|\cos\psi\right| > \frac{\Delta p_A}{\gamma\, p} = \mu$$

- The Möller cross-section can now be integrated within this limits obtaining:

$$\sigma_{Tousch.} = \frac{8\pi r_0^2}{\beta^4}\left[\frac{1}{\mu^2} - 1 + \ln\mu\right]$$

- After some additional algebra and assuming gaussian distributions, we finally obtain the Touschek lifetime for a flat beam:

$$\boxed{\frac{1}{\tau_{Tousch.}} = \frac{\sqrt{\pi}\, r_0^2 c}{\gamma^3}\,\frac{N}{(4\pi)^{3/2}\sigma_X\sigma_Y\sigma_S\,\sigma'_X}\,\frac{1}{\left(\Delta p_A/p_0\right)^2}\,C(\zeta_T)}$$

**with**

$$\zeta_T = \left(\frac{\Delta p_A}{\gamma p_0 \sigma'_X}\right)^2$$

**and**

$$C(\zeta_T) = \zeta_T \int_{\varepsilon_T}^{\infty} \frac{1}{u^2}\left[\frac{u}{\zeta_T} - \frac{1}{2}\ln\left(\frac{u}{\zeta_T}\right) - 1\right] e^{-u}\, du \sim \ln\left(\frac{1}{1.78\zeta_T}\right) - 1.5$$

and where the approximate expression can be used for $\zeta_T < 1$.

- A similar equation can be obtained for the case of round beams.

# Minimizing Touschek Losses

$$\frac{1}{\tau_{Tousch.}} = \frac{\sqrt{\pi} r_0^2 c}{\gamma^3} \frac{N}{(4\pi)^{3/2} \sigma_X \sigma_Y \sigma_S \, \sigma'_X} \frac{1}{(\Delta p_A / p_0)^2} C(\zeta_T)$$

- **In many electron (and positron) storage rings, high current and small emittances are usually required. This makes of the Touschek effect the major responsible for particle losses in such rings.**
- **Depending on the application, a tradeoff between the different requirements must be defined.**
- **In most colliders, the energy is usually fixed, larger emittances (and thus larger beam sizes) are welcome while the current should be as high as possible. Short bunches are preferred (hourglass effect).**
- **In synchrotron light sources, higher beam energy and longer bunches (harmonic cavities) can be used, while emittances (and beam sizes) must be small and the current must be high.**
- **For all applications, the momentum acceptance must be maximized** ⇒

$$\frac{\Delta p_A^2}{p_0^2} \propto \hat{V}_{RF} \qquad RF \quad \text{Acceptance}$$

- **Dynamic aperture**

# Quantum Lifetime

**At a fixed observation point along a storage ring, the transverse motion of a particle is sampled as a pure sinusoidal oscillation:**

$$x_T = a\sqrt{\beta_T}\,\sin\left(\omega_{\beta_T} t + \varphi\right) \qquad T = x, y$$

**Usually, tunes are chosen in order to avoid resonances. In such a situation at a fixed azimuthal position, a particle turn after turn sweeps all possible positions between the envelope:** $\pm a\sqrt{\beta_T}$

**In the presence of synchrotron radiation, photon emission randomly changes the "invariant" $a$ and consequently changes the trajectory envelope as well.**

**Cumulative photon emission can bring the particle envelope beyond the ring acceptance in some azimuthal point and the particle is lost.**

**The explained loss mechanism is responsible for the so-called transverse quantum lifetime.**

**Similar arguments apply also for the longitudinal plane and the longitudinal quantum lifetime can be defined as well.**

# Quantum Lifetime

- **Quantum lifetime was first estimated by Bruck and Sands:**

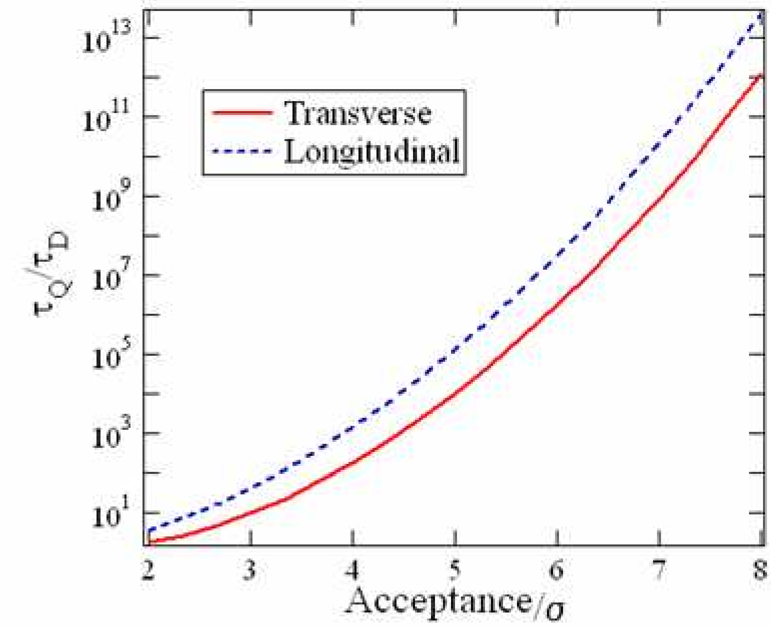
$$\tau_{Q_T} \cong \tau_{D_T} \frac{\sigma_T^2}{A_T^2} \exp\left(A_T^2/2\sigma_T^2\right) \qquad T = x, y$$

*Transverse quantum lifetime*

$$where \quad \sigma_T^2 = \beta_T \varepsilon_T + \left(\eta_T \frac{\sigma_E}{E_0}\right)^2 \qquad T = x, y$$

$$\tau_{Q_L} \cong \tau_{D_L} \exp\left(\Delta E_A^2/2\sigma_E^2\right)$$

*Longitudinal quantum lifetime*




**For an iso-magnetic ring:**

$$\frac{\Delta E_A^2}{2\sigma_E^2} = \frac{J_L E_0}{\alpha_C h E_1} F\left(\frac{e\hat{V}_{RF}}{U_0}\right) \approx \frac{J_L E_0}{\alpha_C h E_1}\left(2\frac{e\hat{V}_{RF}}{U_0} - \pi\right)$$

$$E_1 \cong 1.08 \times 10^8 \quad eV$$

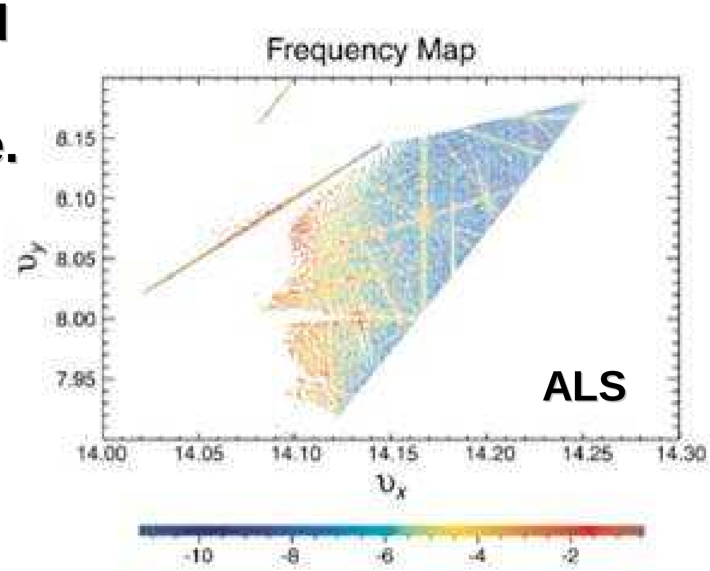
- **Quantum lifetime very strongly depends on the ratio between acceptance and rms size. Values for this ratio of 6 or little larger are usually required.**

- **Transverse quantum lifetime sets the minimum requirement for the transverse aperture, while the longitudinal one defines the minimum momentum acceptance necessary from the lifetime point of view.**

# Tune Resonances

$$l \cdot \nu_x + m \cdot \nu_y = i \qquad l + m \equiv resonance\ \ order \quad and \quad l, m, i \ \ integers$$

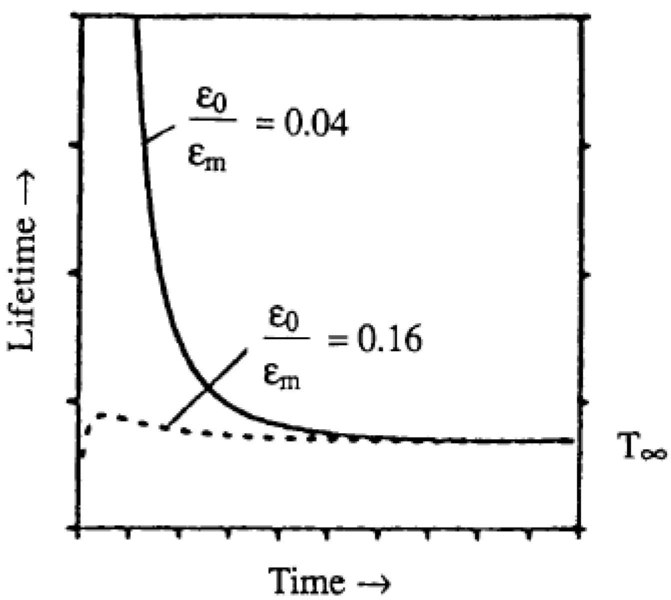
- **Tune resonances are carefully avoided in designing storage rings.**
  - **In fact, particles trapped in a resonance can be quickly lost. Lower order resonances are usually more dangerous. (<~ $12^{th}$ for protons and <~ $4^{th}$ for electrons)**
- **Anyway, imperfections, nonlinear effects and phenomena associated with momentum diffusion can bring the particle on a resonance.**
  - **Examples of common tune shift effects in storage rings: *non linear multipole terms in magnetic fields* (tune shift on amplitude), *beam-beam effects during collision*, "*wakefields*" (tune shift on current), *inelastic gas scattering combined with nonzero chromaticity*, ...**




- **The working point in the tune plane must be carefully selected in order to minimize the impact of all such effects.**

# Remarks on the Lifetime for Protons and Heavy Ions

• **In protons and heavy ions storage rings no damping is present. As a consequence any perturbation on the particle trajectories builds-up and can eventually lead to the loss of the particle.**




• **On the other hand, important loss mechanisms for electrons, become negligible for protons. These include for example, Touschek and gas bremsstrahlung scattering.**

• **Other effects such as *elastic gas scattering*, *molecule excitation*, *fluctuations in the magnetic and RF fields*, *Coulomb scattering (intra-beam scattering)*, …, add up to generate a lifetime of the order of hundreds of hours typically.**

• **Quite often in colliders, the interaction between the colliding beams, the so-called *beam-beam effect*, becomes the main mechanism of losses.**

# References

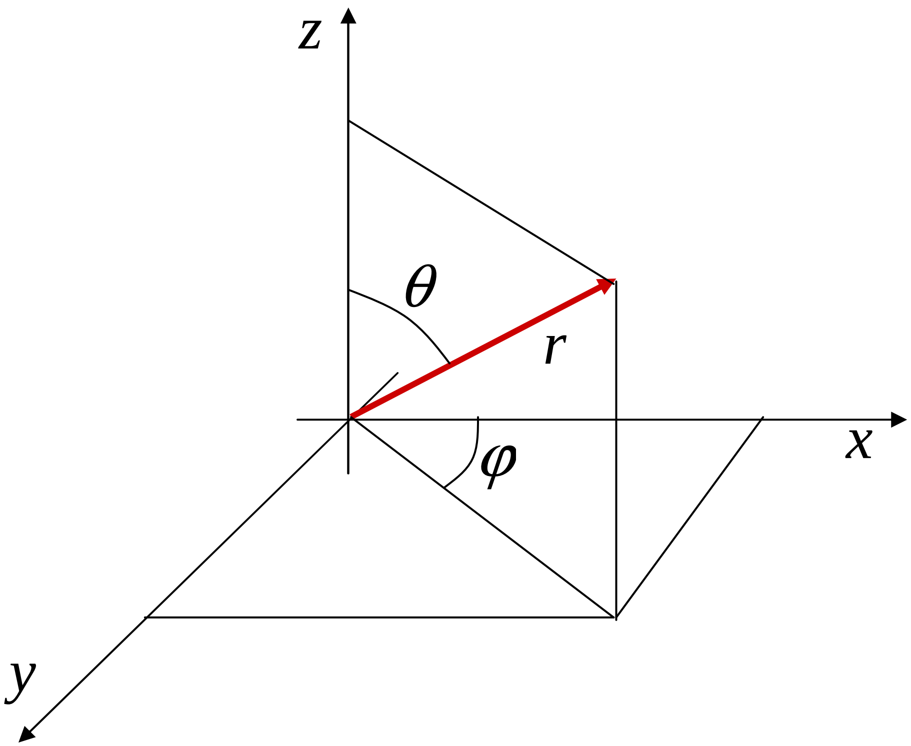

- A. Wrulich*, Single Beam Lifetime,*
  CAS - 5th General accelerator physics course, CERN 94-01

- M. Sands*, The Physics of Electron Storage Rings. An Introduction,*
  SLAC Report 121 UC-28 (ACC) (1970)

# Possible Homework

- Show how different lifetime contributions add up together for the total lifetime. Explain the physical meaning of the constant $\alpha$ (the inverse of the lifetime).
- Estimate the lifetime for the DAΦNE electron beam. Use the line in the lifetime plot in the examples viewgraph.
- Calculate the number of molecules per $cm^3$ for a gas of $N_2$ at the pressure of 1 nTorr.
- Estimate the lifetime due to elastic gas scattering for a 1.9 GeV electron beam at a pressure of 1 nTorr. Assume that the gas is mainly $N_2$ (Z=7), that the average ring beta function is 1.5 m and that the ring acceptance is $10^{-6}$ m. Remember that $\varepsilon_0 = 8.8543 \times 10^{-12}$ F $m^{-1}$.
- For the same ring of the previous problem, calculate the lifetime due to gas bremsstrahlung for the case of 1% relative momentum acceptance.
- For the same ring, calculate also the Touschek lifetime for a bunch current of 10 mA and average rms beam sizes of 100 μm, 10 μm and 1 cm, for x, y and s respectively. Assume an average rms value for x' of 60 μrad.
- Finally, estimate for the same ring the longitudinal quantum lifetime when the longitudinal damping time is 10 ms.

# Spherical Coordinates And Solid Angle




$$x = r\sin\theta\cos\varphi$$

$$y = r\sin\theta\sin\varphi$$

$$z = r\cos\theta$$

$$dx\,dy\,dz = r^2\sin\theta\,dr\,d\varphi\,d\theta$$

$$d\Omega = \sin\theta\,d\varphi\,d\theta$$

$$\int d\Omega = \int_0^{2\pi} d\varphi \int_0^{\pi} \sin\theta\ d\theta = 4\pi$$